

Tutorial session 1

Sept 2026

1. Find the close form and the generating function for the sequences
$$a_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ 5a_{n-1} - 6a_{n-2} & \text{if } n > 1; \end{cases} \quad b_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ -b_{n-1} + 6b_{n-2} & \text{if } n > 1. \end{cases}$$
2. How many binary sequences n -length sequences $a_1a_2 \dots a_n$ are there consisting of 0's and 1's, if no two consecutive terms are both 1's. Same question if no three consecutive terms are 1's
3. How many possible ways are there to form n -letter words using only the letters a, b, c ? How many such words consist of three distinct letters?
4. How many different number plates for cars can be made if each number plate contains two letters (from A to Z) followed by five digits from 0 to 9?
5. (The number of binary sequences of length n containing exactly k ones is $\binom{n}{k}$.) How many three-digit numbers abc have the property that $a \leq b \leq c$?
6. Suppose e_n is the number of string of length n with symbols from the alphabet $\{a, b, c, d\}$ with an even number of a 's.
(a) What is e_0 ; (b) Show that $e_n = 2e_{n-1} + 4^{n-1}$, $n \geq 1$; (c) Find the generating function for e_n .
7. Let a_n be the number of sequences of 0s, 1s and 2s of length n in which a 0 can only be followed by a 1. Find the recurrence relation which the sequence a_n satisfies, and the ordinary generating function.
8. In a city, the bicycle rental company assigns each bicycle a registration number consisting of a sequence (possibly empty) of letters from a, b, c , followed by a sequence (possibly empty) of digits from 0 and 1. How many registration numbers of length n are there?