THE EQUIDISTRIBUTION OF SOME DESCENT SET BASED STATISTICS ON WORDS

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For a length $n$ permutation $\pi$, $\text{Des} \pi$ (respectively, $\text{Desc} \pi$) denotes the descent set of $\pi$ (respectively, the set $\{n - i \mid i \in \text{Des} \pi\}$, i.e, the descent set of the reverse-complement of $\pi$), and $\text{Ides} \pi$ denotes the descent set of $\pi^{-1}$; and $\text{Des}$, $\text{Desc}$ and $\text{Ides}$ become set valued statistics. In 1976 Foata and Schützenberger showed that the bistatistics $(\text{Des}, \text{Ides})$ and $(\text{Desc}, \text{Ides})$ have the same distribution on the set of same-length permutations. Their proof uses the Robinson-Schensted correspondence between permutations and ordered pairs of standard Young tableaux, and they asked for a proof that could avoid the use of that correspondence. In this presentation such a proof is given, and extending $\text{Ides}$ to words we show that $(\text{Des}, \text{Ides})$ and $(\text{Desc}, \text{Ides})$ have the same distribution on the set of rearrangements of the symbols of a word.

As a consequence, we show the joint equidistribution on the rearrangements of the symbols of a word of $\text{stat}$, $\text{maj}$ and $\text{Ides}$, and of $\text{maj}$, $\text{stat}$ and $\text{Ides}$, together with other statistics; here $\text{maj}$ is the celebrated major index statistic, and $\text{stat}$ is the generalization given by Kitaev and the present author (2016) of a Mahonian statistic which is defined originally on permutations in terms of vincular patterns by Babson and Steingrimsson (2000). This equidistribution is a generalization from permutations to words of a result of Burnstein (2010), and on which our construction is also based, and it refines a result stated in the above mentioned 2016 paper.