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The equidistribution of some descent set based statistics on words

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For a length n permutation π, $\mathsf{Des}\,\pi$ (respectively, $\mathsf{Desrc}\,\pi$) denotes the descent set of π (respectively, the set $\{n-i \mid i \in \mathsf{Des}\,\pi\}$, i.e, the descent set of the reverse-complement of π), and $\mathsf{Ides}\,\pi$ denotes the descent set of π^{-1}; and Des, Desrc and Ides become set valued statistics. In 1976 Foata and Schützenberger showed that the bistatistics $(\mathsf{Des}, \mathsf{Ides})$ and $(\mathsf{Desrc}, \mathsf{Ides})$ have the same distribution on the set of same-length permutations. Their proof uses the Robinson-Schensted correspondence between permutations and ordered pairs of standard Young tableaux, and they asked for a proof that could avoid the use of that correspondence. In this presentation such a proof is given, and extending Ides to words we show that $(\mathsf{Des}, \mathsf{Ides})$ and $(\mathsf{Desrc}, \mathsf{Ides})$ have the same distribution on the set of rearrangements of the symbols of a word.

As a consequence, we show the joint equidistribution on the rearrangements of the symbols of a word of stat, maj and Ides, and of maj, stat and Ides, together with other statistics; here maj is the celebrated major index statistic, and stat is the generalization given by Kitaev and the present author (2016) of a Mahonian statistic which is defined originally on permutations in terms of vincular patterns by Babson and Steingrímsson (2000). This equidistribution is a generalization from permutations to words of a result of Burnstein (2010), and on which our construction is also based, and it refines a result stated in the above mentioned 2016 paper.